The central surface density of "dark halos" predicted by MOND

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ABSTRACT

Prompted by the recent claim, by Donato & al., of a quasi-universal central surface density of galaxy dark matter halos, I look at what MOND has to say on the subject. MOND, indeed, predicts a quasi-universal value of this quantity for objects of all masses and of any internal structure, provided they are mostly in the Newtonian regime; i.e., that their mean acceleration is at or above a_0. The predicted value is $\gamma\Sigma_M$, with $\Sigma_M \equiv a_0/2\pi G = 138(a_0/1.2 \times 10^{-8}\mathrm{cm\ s}^{-2})M_\odot\mathrm{pc}^{-2}$, and γ a constant of order 1 that depends only on the form of the MOND interpolating function. For the nominal value of a_0, $\log(\Sigma_M/M_\odot\mathrm{pc}^{-2}) = 2.14$, which is consistent with that found by Doanato & al. of 2.15 ± 0.2.

MOND predicts, on the other hand, that this quasi-universal value is not shared by objects with much lower mean accelerations. It permits halo central surface densities that are arbitrarily small, if the mean acceleration inside the object is small enough. However, for such low-surface-density objects, MOND predicts a halo surface density that scales as the square root of the baryonic one, and so the range of the former is much compressed relative to the latter. This explains, in part, the finding of Donato & al. that the universal value applies to low acceleration systems as well. Looking at literature results for a number of the lowest surface-density disk galaxies with rotation-curve analysis, I find that, indeed, their halo surface densities are systematically lower then the above "universal" value.

The prediction of Σ_M as an upper limit, and accumulation value, of halo central surface densities, pertains, unlike most other MOND predictions, to a pure "halo" property, not to a relation between baryonic and "dark matter" properties.

Subject headings: galaxies: kinematics and dynamics; cosmology: dark matter, theory.

1. introduction

Donato & al. (2009) have recently looked at the distribution of the central surface densities, Σ_c, of the dark matter halos (hereafter CHSD) of galaxies of different types. They find that the distribution is rather narrow, with a central value $\Sigma_c = 10^{2.15\pm0.2}M_\odot\mathrm{pc}^{-2}$. This finding agrees with previous studies, in particular with that of Milgrom & Sanders 2005, who dealt with the relevance to MOND, and with others (see references in Donato & al. 2009). Σ_c is defined by Donato &

al. as the product of the central halo density, ρ_0, and the core radius, r_0, both derived by fitting halo-plus-baryons models to various observations, such as rotation curves, weak lensing results, or velocity dispersion data. In deducing ρ_0 and r_0, the halo is sometimes assumed to have a density distribution of the cored isothermal form; Donato & al. assumed a spherical Burkert profile.

A surface density of special role, Σ_c, translates into an acceleration of a special role $\Sigma_c G$, and this immediately evokes MOND. One is thus naturally led to consider whether such a special value for the CHSD is predicted by MOND.

Brada & Milgrom (1999) showed that MOND predicts an absolute acceleration maximum, of order a_0, that any phantom halo can produce, anywhere in an object. Milgrom & Sanders (2005), in a precursor to Donato & al. (2009), tested this MOND prediction by plotting, for a sample of 17 Ursa Major galaxies, the deduced ρ_0 and r_0 against each other. (These were deduced for a cored isothermal sphere model, not a Burkert one, with a variety of assumptions on the stellar M/L values: maximum disc, population synthesis values, best MOND fits to rotation curves, etc.) They found, for their sample, that these parameters lie near a line of constant $\Sigma_c = 10^2 M_\odot \mathrm{pc}^{-2}$ (their Fig.4), in agreement with the value Donato & al. find. This was interpreted by Milgrom & Sanders (2005) as indicating a maximum halo acceleration as suggested by Brada & Milgrom (1999), because the sample used was devoid of truly low-surface brightness galaxies, for which "halo" accelerations are supposedly lower.

Here I will show, as a new result, that MOND does indeed predict a quasi-universal value for the CHSD of the imaginary, or phantom, dark matter (DM), but only for baryonic systems that are, by and large, in the Newtonian regime, having mean internal accelerations of order a_0 or larger. In contradistinction, MOND predicts that, in principle, we can have galaxies with arbitrarily small values of Σ_c, if the baryonic surface density is low enough. However, the predicted CHSD scales as the square root of the baryonic surface density, and so will have a rather contracted span in a given sample.

Of course, each of the objects in the sample studied can, and should, be used to subject MOND to a detailed, individual test. Inasmuch as MOND passes theses tests, as it seems to do quite well, we can deduce that there is an acceptable halo model whose analog of Σ_c agrees with the MOND prediction. If other halo models do not agree with the MOND prediction, it only shows that there is a range of acceptable halo parameters, within the uncertainties in the model parameters or assumptions (assumed density law for the halo, stellar M/L values, etc.).

Individual tests are, collectively, more decisive than tests of general rules, which they subsume. Nevertheless, deducing and testing such general rules, such as the mass-rotational-speed relation (aka the baryonic Tully-Fisher relation), or the MOND prediction underlying the Faber-Jackson relation, have obvious merits of their own, as they focus attention on certain unifying principles. In this light it is important to consider the prediction of a quasi-universal CHSD in itself.

In section 2, I explain how the quasi-universal CHSD arises in MOND, for high-acceleration systems. In section 3, I treat systems with low surface density; in particular, I show from results

in the literature that disk galaxies with the lowest surface densities analyzed to date, do have Σ_c values that fall systematically below the quasi-universal value. The discussion section 4 deals with the special significance of the prediction at hand, in comparison with other MOND predictions.

2. The emergence of a quasi-universal "halo" central surface density in high acceleration systems

I shall be using the formulation of MOND as modified gravity put forth by Bekenstein & Milgrom (1984). In this theory the MOND gravitational potential, ϕ, is determined by a nonlinear generalization of the Poisson equation

$$\vec{\nabla} \cdot [\mu(|\vec{\nabla}\phi|/a_0)\vec{\nabla}\phi] = 4\pi G\rho, \tag{1}$$

ρ being the true ("baryonic") matter density. Here $\mu(x)$ is the interpolating function characterizing the theory, and a_0 is the MOND acceleration constant, known from various analyses to be $a_0 \approx 1.2 \times 10^{-8}$cm s^{-2} (see, e.g., Stark, McGaugh, & Swaters 2009 who find that gas dominated galaxies satisfy the mass-asymptotic-rotational-velocity relation predicted by MOND, $M = a_0^{-1}G^{-1}V_\infty^4$, with this value of a_0). Similar results will follow from the pristine, algebraic formulation of MOND (Milgrom 1983). Also, if the halo properties are derived from rotation-curve analysis, the same results will follow in modified inertia theories, since these theories predict the algebraic relation between the Newtonian and MOND accelerations for circular orbits. We do not know exactly what these modified inertia theories say about gravitational lensing, but we expect similar results from this as well. Regarding lensing, the existing relativistic extension of the modified-Poisson theory, TeVeS (see Bekenstein 2006 and Skordis 2009 for reviews), says that we can use the halo as deduced from the modified Poisson theory to derive lensing in the standard way, at least when we can assume approximate spherical symmetry. Weak-lensing halo properties can thus be compared directly with the predictions of this theory.

When interpreted by a Newtonist, the departure predicted by MOND, and encapsulated in the difference between the MOND acceleration field $\vec{\nabla}\phi$ and the Newtonian one, is explained by the presence of "dark matter", or "phantom matter" whose density is (Milgrom 1986)

$$\rho_p = \frac{1}{4\pi G}\Delta\phi - \rho. \tag{2}$$

Using the field equation (1) we can write

$$\rho_p = -\frac{1}{4\pi Ga_0}(\mu'/\mu)\vec{\nabla}|\vec{\nabla}\phi| \cdot \vec{\nabla}\phi + (\mu^{-1} - 1)\rho, \tag{3}$$

which can be cast in another form

$$\rho_p = -\frac{a_0}{4\pi G}\mathbf{e} \cdot \vec{\nabla}\mathcal{U}(|\vec{\nabla}\phi|/a_0) + (\mu^{-1} - 1)\rho, \tag{4}$$

where $\mathcal{U}(x) = \int L(x)dx$, with $L = x\mu'/\mu$ the logarithmic derivative of μ, and $\mathbf{e}$ is a unit vector in the direction of $\vec{\nabla}\phi$. This form is particularly useful for calculating column densities of ρ_p along field lines, as we want to do here. This relation is exact. Expression (4), with $\vec{\nabla}\phi$ replaced by $-\mathbf{g}$, holds exactly in the more primitive, algebraic formulation, whereby the MOND acceleration $\mathbf{g}$ is given by $\mu(|\mathbf{g}|/a_0)\mathbf{g} = \mathbf{g}_N$; $\mathbf{g}_N$ being the Newtonian acceleration; $\mathbf{g}$ is not generally derivable from a potential.

Consider first an arbitrary point mass, and integrate expression (4) along a line through the point mass. This gives the central surface density of the phantom matter halo surrounding the mass, $\Sigma(0)$. Inside the mass $\mu \approx 1$ so the second term does not contribute. The integral is performed in two segments: from $-\infty$ to the point mass (where $\mathbf{e}$ is opposite the direction of integration) and from the other side of the point mass to ∞. The two combined give

$$\Sigma(0) = \int_{-\infty}^{\infty} \rho_p dz = \Sigma_M[\mathcal{U}(\infty) - \mathcal{U}(0)] = \Sigma_M \int_0^{\infty} L(x)dx \equiv \lambda\Sigma_M, \tag{5}$$

where,

$$\Sigma_M \equiv \frac{a_0}{2\pi G} \tag{6}$$

is the relevant surface density proxy for a_0 in the present context. In the deep MOND regime $(x \ll 1)$ $L(x) \approx 1$, and far outside the MOND regime $L(x) \approx 0$; so λ is of order 1, and depends only on the interpolating function $\mu(x)$.

I am dealing all along with central column density $\Sigma(0) = 2\int_0^{\infty} \rho dr$ of the MOND phantom halo. For a Burkert halo this column density is related to the quantity Σ_c, used by Donato & al., by $\Sigma(0) = (\pi/2)\Sigma_c$. So, translating the column density to the MOND analog of Σ_c, call it Σ_c^*,

$$\Sigma_c^* = (2\lambda/\pi)\Sigma_M \equiv \gamma\Sigma_M. \tag{7}$$

We have

$$\Sigma_M = 138(a_0/1.2 \times 10^{-8}\text{cm s}^{-2})M_{\odot}\text{pc}^{-2}, \tag{8}$$

or, for the nominal value of a_0, $\log(\Sigma_M/M_{\odot}\text{pc}^{-2}) = 2.14$, compared with the value $\log(\Sigma_c/M_{\odot}\text{pc}^{-2}) = 2.15 \pm 0.2$ found by Donato et al.[1].

For the limiting form of $\mu(x)$–with $\mu(x) = x$, for $x \leq 1$, and $\mu(x) = 1$, for $x > 1$–we have $\lambda = 1$, and $\gamma = 2/\pi$. For $\mu(x) = x(1 + x^2)^{-1/2}$, we have $\lambda = \pi/2$, and $\gamma = 1$. Values of λ for other forms of μ can be read off Fig. 3 of Milgrom & Sanders (2008) (where they were deduced numerically,

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[1]The predicted MOND "halo" of an isolated system is not well described by a Burkert profile: The MOND "halo" density behaves asymptotically as r^{-2}, not r^{-3}, and it is expected to have a depression around the center not a decreasing density profile everywhere. Nevertheless, these differences are expected to produce only differences by a factor of order 1 in the resulting Σ_c. The very near equality of Σ_M and the central value found by Donato & al. is thus somewhat fortuitous.

and appear for other purposes). One sees that $1 \lesssim \lambda \lesssim 3$, and so $0.7 \lesssim \gamma \lesssim 2$ for the range of μ forms studied there[2].

Equations (5)-(7) are our basic result, around which all else in the paper revolves. They tell us that for the simple case of a point mass a universal value of Σ_c is indeed predicted by MOND; its value is $\approx \Sigma_M$, which agrees very well with the value found by Donato & al..

Consider now an extended mass, M. If the mass is well contained within its MOND transition radius, $R_M = (MG/a_0)^{1/2}$, namely if the Newtonian accelerations, and hence the MOND accelerations, are high everywhere within the mass, then the procedure we followed for a point mass applied approximately, and we get again $\Sigma_c^* \approx \gamma \Sigma_M$.

Here I have to pause, and comment on a subtlety in the use of eq.(4), and in interpreting the results thereof. This I demonstrate with two examples. First consider a mass of finite extent whose density does not increase towards its center as r^{-1} or faster. In this case, the Newtonian acceleration, and so also the MOND acceleration, goes to zero at the center, even if these accelerations are much higher than a_0 in most of the bulk. In other words, there are two MOND regimes: one within some small sphere of radius r_1 around the center, and another beyond the MOND transition radius, $R_M = (MG/a_0)^{1/2}$. The small r region contributes to $\Sigma(0)$ through the first term in eq.(4), an amount $-\Sigma_M \int_0^{X_0} L(x)dx$, where X_0 is the maximum (MOND) acceleration in units of a_0. This contribution is $\approx -\lambda \Sigma_M$ for $X_0 \gg 1$. The outer region contributes a positive quantity of the same magnitude. In addition, the inner region contributes through the second term in eq.(4), and its total contribution is positive (the phantom density is always positive in the spherical case). The inner region of phantom mass, even if it contributes to $\Sigma(0)$, has only little mass, is dynamically unimportant, at large, and should not be included when comparing with results for global halo parameters. I shall thus ignore it, and take $\lambda \approx \int_0^{X_0} L(x)dx$. When the baryonic surface density is low, the central, low-acceleration region is expanded and encompasses the whole mass. The contribution of the first term in eq.(4) then can, indeed, be taken to vanish, and the contribution to $\Sigma(0)$ comes from the second term.

In another example, consider two arbitrary point masses along the line of sight. Integrating the phantom density in eq.(4) along the line of sight now gives $\Sigma(0) = 2\lambda \Sigma_M$. (We now have to integrate over four segments over which $\mathbf{e}$ changes sign: from $-\infty$ to the first mass, from there to the zero-field point somewhere between the masses, from there to the second mass, and from there to infinity). This value is exact and independent of the distance between the masses. How is this consistent then with our deduction that $\Sigma(0) \approx \lambda \Sigma_M$ for all systems well within their transition radius? When the two masses are well separated, by more then their joint transition radius, there is an extended halo surrounding each of the masses, each halo with its own $\Sigma(0) \approx \lambda \Sigma_M$, and the two column densities add up. When the two masses are near each other, well within their joint

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[2]The coefficient λ diverges if $1 - \mu(x)$ behaves at large x as x^{-1} or slower. The divergence does not occur in the MOND regime, but comes from the Newtonian regime very near the point mass. Such a behavior of μ is, however excluded strongly from solar system constraints, and I preclude it.

R_M, there will be a common halo of phantom matter residing roughly beyond R_M, and this indeed has $\Sigma(0) \approx \lambda \Sigma_M$ [arising from integrating eq(4) in the outer two segments]. In addition, there is a small region around the point of zero field between the two masses, which contributes the same amount to the central column density, but which contains little mass, is dynamically unimportant in the present context, and should be eliminated from the result that is to be compared with the observations.

Keeping these caveats in mind, the reasoning leading to eq.(5) can be applied not only to spherical systems. For example, for a disk galaxy with a high central surface (baryonic) density, $\Sigma_b(0) \gg \Sigma_M$, we can use this equation to calculate the column density either along the symmetry axis, or along a diameter in the plane of the disc (in both cases the field is always parallel or antiparallel to the line of integration). If we ignore the small region of phantom matter near the very center (or if we add a small matter cusp that prevents the acceleration from vanishing at the center) we again get $\Sigma(0) = \lambda \Sigma_M$.

Take now, more generally the extent of our mass to be R, and its mean density ρ, and define $\Sigma_b = \rho R$, the baryonic equivalent of Σ_c. The second term in eq.(4) can be estimated to contribute to $\Sigma(0)$

$$\approx 2\rho R[\mu^{-1}(g/a_0) - 1], \tag{9}$$

where g is the MOND mean acceleration inside the mass, and is given by

$$(g/a_0)\mu(g/a_0) \approx \frac{4\pi}{3}\frac{\rho RG}{a_0} = \frac{2}{3}\frac{\Sigma_b}{\Sigma_M}. \tag{10}$$

The first term in eq.(4) is taken to contribute $\approx \Sigma_M \int_0^{X_0} L(x)dx$, where $X_0 = g/a_0$. Thus, we can write

$$\Sigma_c^* = (2/\pi)\Sigma(0) \approx \Sigma_M\{(6/\pi)X_0[1 - \mu(X_0)] + \int_0^{X_0} L(x)dx\}. \tag{11}$$

For $X_0 \gg 1$ this gives $\Sigma_c^* \approx \gamma \Sigma_M$, again.

3. Low surface density systems

MOND does permit arbitrarily low values of Σ_c for phantom halos in low acceleration systems. When $X_0 \ll 1$, namely, when the maximum (MOND) acceleration in the system is much smaller than a_0, we get from eq.(11), to lowest order in X_0,

$$\Sigma_c^* \approx (6/\pi + 1)\Sigma_M X_0 \approx 2.4 \left(\frac{\Sigma_b}{\Sigma_M}\right)^{1/2} \Sigma_M. \tag{12}$$

Such low acceleration systems are characterized by low baryonic surface densities $\Sigma_b/\Sigma_M \ll 1$. Note, however, that the departure from the universal Σ_c^* sets in at rather low baryonic surface densities, since Σ_c^*/Σ_M scales as the square root of Σ_b/Σ_M. The lowest acceleration disc galaxies

studied to date have X_0 values only down to 0.1-0.2; and we see from eq.(12) that even for values of X_0 as low as $1/5$ we get $\Sigma_c^* \approx 0.6\Sigma_M$. Clearly, however, MOND does predicts that, for extremely low baryonic surface density galaxies, the CHSD falls increasingly below the quasi-universal value.

To superficially check this expectation, I looked (rather randomly) in the literature for derived halo parameters for the lowest acceleration disk galaxies with rotation-curve analysis. Three such galaxies were analyzed in light of MOND by Milgrom & Sanders (2007), showing rather satisfactory agreement. These were also analyzed earlier in terms of cored isothermal halos: For KK98 250 and KK98 251, I find in Begum & Chengalur (2005) best-fit parameters that give $\Sigma_c = 56$, and $66M_\odot/\mathrm{pc}^2$, respectively. For NGC 3741, Begum & al. (2005) find parameters that yield $\Sigma_c = 56M_\odot/\mathrm{pc}^2$. All three values fall substantially below the nominal quasi-universal value of $\Sigma_c = 140M_\odot/\mathrm{pc}^2$, and are consistent, within the uncertainties, with our rough estimate (12), having X_0 values of between 0.1 and 0.3. The first two galaxies were not included in the Donato & al. analysis; but NGC 3741 was included, based on the analysis of Gentile & al. (2007) (assuming a Burkert, not a cored isothermal halo), whose results give $\Sigma_c = 74$. This value is higher than the result of Begum & al. (2005) (though consistent within the uncertainties), but still only about half the quasi-universal value.

Another low acceleration galaxy that is worth analyzing in detail (and is not included in the Donato & al. sample) is the dwarf Andromeda IV. Its rotation curve is given in Chengalur & al. (2007). To my knowledge, its photometry and HI distribution are not yet available publicly for rotation curve analysis. However, according to Chengalur et &. (2007) it is heavily dominated by gas with $M_{gas}/L \approx 18$, and it shows a very strong mass discrepancy with $M_{dyn}/M_{gas} \approx 14$ at the last measured point. In deriving a cored isothermal halo parameters we can thus approximately ignore the baryons and fit the rotation curve with the halo alone. Doing this, I find, tentatively, $\Sigma_c \sim 45M_\odot/\mathrm{pc}^2$, about three times lower than Σ_M. Since in this case $X_0 \sim 0.1 - 0.15$, this is also in agreement with the estimate of eq.(12).

Why then do Donato & al. suggest that the quasi-universal value of Σ_c applies to all galaxies, including the very low-acceleration ones? This is based mostly on the analysis of dwarf spheroidal satellites of the Galaxy. Their analysis includes only one well studied low-acceleration disk, the above mentioned, NGC 3741–for which, as we saw, the actual Σ_c could be lower–and one somewhat higher acceleration galaxy, DDO 47. As regards the dwarf spheroidal Milky-Way satellites, MOND would indeed predict lower values of Σ_c than adopted by Donato & al.. However, as Donato & al. emphasize themselves, the analysis of these systems is beset by uncertainties in the model assumptions (e.g., assumptions on orbital anisotropies), leading to non-unique results. Angus (2008) has analyzed these dwarf spheroidals in MOND, and found that, with two exceptions perhaps, they can be well explained by MOND, assuming appropriate orbit anisotropy distributions. This would mean, as I stressed above, that there are acceptable "halo" models that are consistent with the predictions of MOND. The disparate values adopted by Donato & al. only demonstrate the non-uniqueness of the halo-parameter determination.

4. Discussion

I have shown that the acceleration constant of MOND a_0 defines a special surface density parameter $\Sigma_M = a_0/2\pi G$. This serves as a quasi-universal central surface density of phantom halos around objects of all masses and structures, provided they are themselves in the Newtonian regime (i.e., with bulk accelerations of order a_0 or higher).

This is a particularly interesting prediction of MOND, because most of the other salient MOND predictions relate properties of the true matter (baryons) to those of the putative dark matter halo. This is the case for the mass-velocity (baryonic Tully-Fisher) relation, the Faber-Jackson relation, the transition from baryon dominance to DM dominance at a fixed acceleration, the full prediction of rotation curves, the necessity of a disk component of DM, in disk galaxies, in addition to a spheroidal halo, etc. (see Milgrom 2008 for a more detailed list, and explanations). Here, however, we have a prediction that speaks of a property of halos themselves, without regard to the true mass that engenders them, apart from the requirement that the baryons be well concentrated. Σ_M may also be viewed as an upper limit, and accumulation value, for "halo" central surface densities, irrespective of baryonic properties. It is clear then, that the a_0 that appears in this prediction need have nothing to do with the a_0 that appears in other relations, in the framework of the DM doctrine. We could have a sample of halos all satisfying the present prediction, and add to them baryons arbitrarily, so as not to satisfy, e.g., the baryonic-mass-velocity relation $MGa_0 = V^4$, which also revolves around some acceleration constant. The fact that the a_0 emerging from the phenomenology here is the same as that appearing in the other phenomenological relation should be viewed as another triumph of MOND.

There are two other MOND predictions that speak of properties of the halo alone. The first is that the density profile of the "halo" of any isolated object behaves asymptotically as r^{-2} (asymptotic flatness of rotation curves). The other such prediction is the maximally allowed acceleration (of order a_0) that a halo can produce (Brada & Milgrom 1999). This is simply a reflection of the MOND tenet that the phantom mass cannot be present where accelerations are higher than roughly a_0. The prediction I discuss here can be understood, qualitatively, as a result of the above two: On the asymptotic, r^{-2}, tail of the "halo", the acceleration it produces is $g_h \approx 4\pi G\rho r$. Going inward, the maximum-acceleration prediction tells us that this behavior can continue only down to a radius where $g_h \sim a_0$. Below this radius the halo density profile must become shallow and produce a core. This gives $\rho_0 r_0 \sim a_0/4\pi G$. It is this that underlies our more quantitative result here.

However, there is nothing in MOND to forbid the halo density profile from becoming shallow within a radius much larger then that where $g_h \sim a_0$. This can happen at arbitrarily large radii, producing arbitrarily small values of Σ_c^*, as indeed our detailed analysis shows.

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